**Applied Finance Project:**

**Interest Rate Swaption Pricing and Model Calibration Under G2++**

**for Mizuho Securities USA**

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# Executive Summary

The fixed income desk in Mizuho Securities USA (the “company”) implemented the G2++ model in interest rate modelling and utilized such model to price USD swaptions and hedge credit risk; the way how this is achieved is to calibrate model parameters by feeding in market data and to minimize difference between model price and market price.

Our team is dedicated in supporting the firm in model development and calibration. We first examined, the Hull White model, a widely used interest rate model to understand generality in interest rate modelling and how it links to swaption pricing. Specifically asked by the company, we further expanded our research to the G2++ model, a next generation cutting-edge model and perform calibration on model parameters using market swaption data. Eventually, we determined the G2++ model has inconsiderable advantages over the Hull White model in the sense of reflecting the given volatility surface much better and, at last, resulting in less pricing error. However, the existence of the Hull White model in the market practice also proved its unbeatable advantage in implementation: simple and fast!

# Introduction and Background

Mizuho Securities USA’s Fixed Income Division (as the company) provides investment products including US Treasuries, corporate bonds, Debt Capital Markets, US Agency Securities, Securities Financing, Strategic Credit Group and ABS securities. The USD swaption, as the financial product of our research object, consists of ~6% of the interest rate market in notional. It is a right agreed between counterparties, often referred as payer and receiver, upon interest rate terms to be exchanged in the future date, referred as maturity, for a period of time, referred as tenor.

Since most of these trades are done over-the-counter (“OTC”), it is crucial to the business to own an in-house modelling and pricing framework. As per specifically requested by the company, we are pleased to develop a calibration model for USD swaption using the next-generation interest rate model, G2++ model in the hope for helping the company for better pricing and risk management.

Prior to the Financial crisis of 2008, financial markets regarded large, established derivative counterparties as robust and too-big-to-fail. Consequently, investors either underweighted or completely neglected counterparty credit risk. Following the crisis, financial institutions have been increasingly meticulous on evaluating and managing counterparty risk, especially on over the counter (OTC) derivatives traded with un- or semi-collateralized counterparties.

Credit Value Adjustment (CVA) is the market value of counterparty credit risk, and it serves as a handy metric when market makers want to assess counterparty credit charges in addition to risk-free prices when pricing derivatives such as interest rates swap, cross currency swap, and options by gathering firm-wide credit risk and compute deal-by-deal credit charges. CVA desks’ pricing models are often simpler than market-makers’. For instance, short-rate interest rate models are more widely used than LIBOR Market Model because short-rate models have fewer parameters, which shaves off computational cost, thus allowing the firms to quick respond to clients’ deal requests.

Significant adjustments of input parameters with a minor market change should be avoided as this leads to the dramatic, unintentional change of the model’s swaption surface and X-value adjustment (XVA) prices. This will have further implications as the clients are sensitive to the price and unreasonable XVA change makes it difficult to operate appropriate hedging activities.

Consequently, our motivation is to find an appropriate model to simulate the movement of the swaption market in order to better manage risk exposure and avoid pricing anomaly.

This is precisely the reason that we focused on the comparison of two calibration approaches and the respective underlying short-rate models: 1-factor Hull-White model and the G2++ model. Despite the good tractability of the 1-factor Hull-White model, it has two major drawbacks. First, the model is not capable of producing a large volatility surface up to an adequate level because of the lack of free calibration parameters. The G2++ model, on the other hand, is much more flexible in terms of fitting different volatility surfaces attributable to the introduction of a second factor and the mutual correlation parameters. Second, the 1-factor Hull-White model has often been criticized for the intensity of the calibrated negative interest rates which have a major impact on the valuation of the liabilities and the time value of options and guarantees (TVOGs). The general behaviour of the G2++ model is opaquer.

We thus investigate the model behaviour of the 1-factor Hull-White model and the G2++ model, respectively, exploring their analytical solution to pricing swaption. We implement both model approaches, calibrate the models to current market data and analyse the goodness of fit. Furthermore, we established different schemes of “features”, or loss functions in calibration of the models to empirical data.

# Literature Review and Details of Underlying Model

Though the focus of our research lies on the calibration of the G2++ model, we also considered the classic Hull White model as a starting point for its simplicity and going through it shall help us better understand how interest rate model behaves and what benefit we could achieve if moving towards the next level, G2++.

## Hull White Model

As an extension of the Vasicek model, the HW model includes time-dependent parameters, which increases the possibility of calibrating them with respect to market data. The Hull-White model is growing in importance lately due to the reason that it allows the interest rates to be negative while we will not find this feature in many other models. Moreover, the HW model is also widely used in the practice due to simplicity and computational speed.

The Hull White interest rate stochastic process is defined as follow:

, r(0) = 0

Where is the mean reversion constant, is the volatility parameter, and is defined so that the model fits the current term structure:

And F(0,t) is the market instantaneous forward rate from 0 to t:

While the calibration with HW model is quick as only 3 parameters are required to fit the volatility surface, it has often been criticized for the number and intensity of the generated highly negative interest, which, in our case may not be practical in valuating USD interest rate.

## G2++ Model

The G2++ model assumes the instantaneous short-rate process given by the sum of two correlated Gaussian factors plus a deterministic function that is properly chosen so as to exactly fit the current term structure of discount factors. The model is quite analytically tractable in that explicit formulas for discount bonds, European options on pure discount bonds, hence caps and floors, can be readily derived. Gaussian models like this G2++ model, are very useful in practice, despite their unpleasant feature of the theoretical possibility of negative rates. Indeed, their analytical tractability considerably ease the task of pricing exotic products. The Gaussian distribution allows the derivation of explicit formulas for a number of non-plain-vanilla instruments and, combined with the analytical expression for zero-coupon bonds, leads to efficient and fairly fast numerical procedures for pricing any possible payoff.

We assume that the dynamics of the instantaneous-short-rate process under the risk-adjusted measure Q is given by

And the two process and , follow the two stochastic differential equations

Where is a two-dimensional Brownian motion with instantaneous correlation ρ as from , where r0, a, b, σ, η are positive constants, and where −1 ≤ ρ ≤ 1. The function ϕ is deterministic and well defined in the time interval [0, T∗], with T∗ a given time horizon, typically 10, 30 or 50 (years). In particular, ϕ(0) = r0. We denote by Ft the sigma-field generated by the pair (x, y) up to time t.

### Analytical Formula for European Swaption

In our model calibration, we will simply use the close form solution of the pricing of European swaptions at time zero (𝑡=0), as suggested by Brigo and Mercurio (2006 pag. 158) (page. 173-175). The formula writes as the following:

The holder of the swaption has the right to enter the interest rate swap at with payment times , where he pays (receives) at the fixed rate X and receives (pays) LIBOR set “in arrears”.

the year fraction from to ,

for

: the price of the zero-coupon bond, this is provided by discount factor from Bloomberg terminal.

,

Where

,

And

As suggested by Ferranti Matteo in his research paper, boundaries for the integration region of the swaption analytical formula is restricted to:

### 

### Benchmark Model (Normal Model) for European Swaption

As suggested by the Mizuho, the normal model is an alternative benchmark model for swaption pricing, on which the Bloomberg market quote is based, and the derivation of implied volatility quotes. The rational that normal model is chosen over the black model because it is more appropriate for negative yield environment like the investment banks are currently facing. It is hard for black model to reflect the negative yield as logarithm does not give negative value.

Here, the evolution of the forward swap rate is given through the stochastic differential equation (SDE):

Where

We choose “normal market volatility” observed on the market as our benchmark to compare against what the calibrated G2++ is spitting out.

# Data, Calibration and Implementation

## Data Description

Daily data feed into our calibration model is acquired from Bloomberg terminal. It includes daily USD term structure (as USD OIS yield to interpolate continuous discount factor) and daily swaption volatility surface (the sum of volatilities for both payer and receiver). Such volatility surface is what our calibrated model price is trying to compare against. The data below is a snapshot of the European Swaption Volatility surface, its OIS yield curve and Forward Starting Swap.

1. Volatility surface (the data observed is a straddle, meaning the volatility is the sum of the ATM payer and receiver swaption)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tenor | 12M | 24M | 36M | 60M | 84M | 120M | 180M | 240M | 300M | 360M |
| 1M | 74.62 | 79.57 | 85.38 | 56.77 | 60.7 | 68.68 | 57.285 | 57.97 | 60.425 | 56.05 |
| 3M | 35.57 | 72.585 | 82.855 | 60.4 | 63.925 | 69.095 | 59.685 | 59.03 | 58.485 | 57.85 |
| 6M | 70.68 | 79.395 | 78.96 | 60.845 | 68.745 | 65.51 | 64.13 | 62.835 | 62.26 | 61.58 |
| 9M | 56.75 | 64.59 | 65.56 | 69.21 | 69.35 | 66.055 | 65.44 | 64.855 | 64.23 | 63.57 |
| 12M | 71.13 | 71.975 | 74.975 | 71.315 | 69.57 | 65.805 | 64.735 | 61.715 | 61.035 | 65.575 |
| 24M | 70.89 | 77.19 | 77.21 | 75.93 | 76.91 | 74.7 | 62.955 | 70.39 | 59.98 | 68.455 |
| 36M | 76.035 | 77.36 | 76.16 | 77.045 | 76.19 | 75.105 | 62.19 | 57.125 | 59.28 | 69.35 |
| 60M | 81.795 | 80.785 | 79.73 | 77.735 | 76.36 | 74.705 | 71.835 | 68.94 | 68.325 | 68.18 |
| 84M | 71.085 | 78.31 | 77.605 | 76.12 | 76.03 | 73.01 | 71.56 | 68.04 | 67.295 | 66.55 |
| 120M | 76.25 | 72.345 | 72.805 | 72.525 | 73.11 | 71.86 | 68.005 | 66.02 | 64.895 | 62.77 |
| 180M | 69.03 | 67.23 | 65.84 | 63.055 | 63.545 | 63.895 | 60.555 | 57.84 | 56.435 | 55.98 |
| 240M | 63.295 | 61.485 | 60.315 | 58.025 | 58.195 | 56.95 | 55.445 | 52.37 | 52.505 | 51.88 |
| 360M | 57.78 | 55.35 | 56.365 | 54.115 | 54.635 | 52.87 | 53.33 | 52.385 | 51.96 | 51.865 |

* Note the volatility here implied from the market price is under the normal model (7/5/19)

1. OIS term structure, to interpolate continuous discount factor.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Payment Date | Maturity Date | Market Rate | Shift (bp) | Shifted Rate | Zero Rate | Discount | Source |
| 07/10/2019 | 07/10/2019 | 2.42 | 0 | 2.42 | 2.42 | 0.999933 | CASH |
| 07/18/2019 | 07/16/2019 | 2.386449 | 0 | 2.386449 | 2.414586 | 0.999536 | SWAP |
| 07/25/2019 | 07/23/2019 | 2.38705 | 0 | 2.38705 | 2.414634 | 0.999073 | SWAP |
| 08/01/2019 | 07/30/2019 | 2.388999 | 0 | 2.388999 | 2.416061 | 0.998608 | SWAP |
| 08/13/2019 | 08/09/2019 | 2.322 | 0 | 2.322 | 2.346796 | 0.998004 | SWAP |
| 09/11/2019 | 09/09/2019 | 2.229 | 0 | 2.229 | 2.249664 | 0.996176 | SWAP |
| 10/11/2019 | 10/09/2019 | 2.160995 | 0 | 2.160995 | 2.178439 | 0.994508 | SWAP |
| 11/14/2019 | 11/12/2019 | 2.098 | 0 | 2.098 | 2.112335 | 0.992711 | SWAP |
| 12/11/2019 | 12/09/2019 | 2.061 | 0 | 2.061 | 2.073225 | 0.991317 | SWAP |
| 01/13/2020 | 01/09/2020 | 2.015615 | 0 | 2.015615 | 2.025545 | 0.989803 | SWAP |
| 04/13/2020 | 04/09/2020 | 1.920724 | 0 | 1.920724 | 1.925065 | 0.98554 | SWAP |
| 07/13/2020 | 07/09/2020 | 1.849999 | 0 | 1.849999 | 1.849716 | 0.981539 | SWAP |
| 01/13/2021 | 01/11/2021 | 1.733135 | 0 | 1.733135 | 1.734052 | 0.973984 | SWAP |
| 07/13/2021 | 07/09/2021 | 1.662095 | 0 | 1.662095 | 1.660298 | 0.967116 | SWAP |
| 07/13/2022 | 07/11/2022 | 1.594 | 0 | 1.594 | 1.591673 | 0.952978 | SWAP |
| 07/12/2023 | 07/10/2023 | 1.575 | 0 | 1.575 | 1.572781 | 0.938591 | SWAP |
| 07/11/2024 | 07/09/2024 | 1.579 | 0 | 1.579 | 1.577315 | 0.923648 | SWAP |
| 07/13/2026 | 07/09/2026 | 1.638651 | 0 | 1.638651 | 1.639951 | 0.890887 | SWAP |
| 07/11/2029 | 07/09/2029 | 1.745032 | 0 | 1.745032 | 1.753048 | 0.838329 | SWAP |
| 07/11/2031 | 07/09/2031 | 1.81121 | 0 | 1.81121 | 1.824667 | 0.802398 | SWAP |
| 07/12/2034 | 07/10/2034 | 1.880565 | 0 | 1.880565 | 1.900498 | 0.750825 | SWAP |
| 07/13/2039 | 07/11/2039 | 1.945579 | 0 | 1.945579 | 1.972092 | 0.672748 | SWAP |
| 07/13/2044 | 07/11/2044 | 1.967108 | 0 | 1.967108 | 1.993593 | 0.60602 | SWAP |
| 07/13/2049 | 07/09/2049 | 1.973911 | 0 | 1.973911 | 1.998081 | 0.547607 | SWAP |
| 07/11/2059 | 07/09/2059 | 1.956311 | 0 | 1.956311 | 1.967063 | 0.453595 | SWAP |
| 07/11/2069 | 07/09/2069 | 1.923911 | 0 | 1.923911 | 1.914819 | 0.382046 | SWAP |

* This table illustrates the interpolation result of discount factor based on daily OIS interest rate.

1. Forward Starting Swap Rate

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Tenor | 12M | 24M | 36M | 60M | 84M | 120M | 180M | 240M | 300M | 360M |
| 1M | 2.04 | 1.88 | 1.81 | 1.81 | 1.88 | 1.99 | 2.13 | 2.19 | 2.22 | 2.23 |
| 2M | 2.00 | 1.85 | 1.80 | 1.80 | 1.87 | 1.99 | 2.13 | 2.19 | 2.22 | 2.23 |
| 3M | 1.92 | 1.80 | 1.75 | 1.79 | 1.81 | 1.97 | 2.06 | 2.15 | 2.17 | 2.17 |
| 6M | 1.85 | 1.74 | 1.75 | 1.79 | 1.84 | 1.95 | 2.09 | 2.15 | 2.17 | 2.20 |
| 9M | 1.76 | 1.73 | 1.72 | 1.79 | 1.88 | 2.00 | 2.14 | 2.20 | 2.22 | 2.23 |
| 12M | 1.71 | 1.68 | 1.71 | 1.75 | 1.84 | 2.00 | 2.10 | 2.16 | 2.18 | 2.21 |
| 18M | 1.70 | 1.70 | 1.73 | 1.83 | 1.92 | 2.05 | 2.17 | 2.22 | 2.24 | 2.24 |
| 24M | 1.66 | 1.70 | 1.74 | 1.84 | 1.92 | 2.05 | 2.15 | 2.22 | 2.23 | 2.21 |
| 36M | 1.72 | 1.78 | 1.81 | 1.95 | 2.03 | 2.13 | 2.21 | 2.26 | 2.24 | 2.23 |
| 60M | 1.96 | 2.00 | 2.05 | 2.17 | 2.20 | 2.28 | 2.32 | 2.32 | 2.29 | 2.27 |
| 84M | 2.18 | 2.21 | 2.25 | 2.31 | 2.32 | 2.35 | 2.35 | 2.36 | 2.34 | 2.30 |
| 120M | 2.41 | 2.42 | 2.43 | 2.43 | 2.42 | 2.40 | 2.41 | 2.39 | 2.33 | 2.28 |
| 180M | 2.42 | 2.43 | 2.43 | 2.44 | 2.42 | 2.40 | 2.37 | 2.33 | 2.29 | 2.25 |
| 240M | 2.36 | 2.36 | 2.36 | 2.35 | 2.34 | 2.33 | 2.28 | 2.24 | 2.20 | 2.16 |
| 360M | 2.18 | 2.17 | 2.17 | 2.16 | 2.15 | 2.13 | 2.09 | 2.05 | 1.93 | 1.83 |

* This value is needed for the swaption pricing formula as the strike price

Note that OIS (overnight indexed swap) rate is chosen over LIBOR in our calibration to be consistent with the market standard. This is a convention adopted after 2008 financial crisis as LIBOR is no longer viewed as a risk-free asset.

## Calibration Methodology

The calibration of a model may be considered more an art rather than a science; as a matter of facts, despite the usual formulation of the problem as a constrained minimization process of some loss functions, the practitioner may choose a subset of instruments from the market data matrix of caps or swaptions.

For what concerns the present work, we have decided to avoid swaptions with maturity and tenor, respectively, shorter or equal to 1 year to avoid pricing anomaly. Once the calibration instruments have been chosen, it is also possible to assign to each instrument a specific weight; nevertheless, in the present thesis we have decided to give to each instrument the same weight equals to 1.

Following the literature, the first function to be used is the Normal Model formula to pricing swaption, which includes a specific normal volatility parameter. This value is implied by market price (a.k.a. model implied volatility).

On the other side of the equation, G2++ analytical formula is used.

A set of should be found so that the following is obtained:

# Swaption Price Engine Architecture

To work on the project proper, the team has built the entire swaption price engine in Python to serve the purpose of this project. The price engine has been made available on Github and open-sourced to the community. We adopted an object oriented programming approach when designing the solution and decouple each component to perform a major function.

**Loss Function**

**Closed-Form Solution Price**

**Model Implied Volatility**

**Mean Squared Error**

**G2++ Price Engine**

**Initial Model Parameters**

**Forward Starting Swap Rate**

**Interpolation**

**Construct Zero Curve**

**Construct Forward Curve**

**Market Volatility Surface**

**Optimiser**

**Boundary Condition**

**Non-linear Solver**

**OIS Yield Curve**

**Calibrated Parameters**

The interpolation Component constructs the zero curve as well as the forward curve. Although given the G2++ model, only zero curve is required as the calculation of the forward curve is done as part of the analytical solution, forward curve construction method is available to ensure the completeness of the price engine and validation purpose.

The zero curve then feeds into the price engine, together with the data on forward starting swap rate as well as a set of initial parameters for . The price engine will produce the swaption price based on the G2++ closed-form solution, back out the implied volatility and calculate the mean square error between the model and theoretical implied volatility and price from the normal model.

This process will be reiterated by the optimser based on a set of constraints until a minimal mean square error is achieved. The optimizer will produce the calibrated parameter at the end of the process.

# Interpolation of the yield curve

To be able to price the swaption, we first need to construct the yield curve. As previously stated we are referencing the data from OIS USD yield curve from Bloomberg terminal. Interpolation is then done to construct the entire zero curve and the forward curve later on.

The team has looked into the common interpolation methods for the OIS yield curve, including

linear and piecewise cubic. Implementation of both interpolation methods have been incorporated into the price engine from QuantLib which is an open source library that offers yield curve construction. The choice of this open source library was made because it offers customised calendar for each country and allows users to specify the spot rate on the exact date shown in Bloomberg. These features would have been very time consuming to build from scratch. For simplicity, we used linear interpolation for the calibration while piecewise cubic could be used as it is available in the price engine.

# Loss Functions

The team is tasked to run the calibration through 4 different calibration methods and compare results across the board. The 4 loss functions proposed by Mizuho Securities are:

1. The squared difference between model and market prices:
2. The squared difference between model and market prices:
3. The squared percentage difference between model and market prices:
4. The squared percentage difference between model and market prices:

# Optimisation: Boundary Conditions, Algorithm and initial parameters

The boundary condition is provided by Mizuho Securities based their experience and guideline.

Sequential least square quadratic programming is used to perform the optimisation. This is an optimisation algorithm that is available from the Python Scipy library and is known to perform well on optimisation problems with a set of constraint.

The initial parameters are provided as well as a good starting point for the optimisation process. Other initial parameters are experimented as well, and the challenge is that the optimiser will sometimes hit the bounds and exit the process due to error. Such error could be due to a large value which is the power term of the exponential term in the formula. This error could be avoided by setting try and catch statement in the Python code and give a huge penalty so that the optimiser could avoid getting those values.

# Result and Conclusion

The below illustrates the calibrated result of parameters for a given day, under different loss functions. Note that parameters are relatively stable if the loss functions are within the same object of consideration. For example, is 2.81905739 or 2.81905646 when the object of the loss function is chosen to be the price.

1. Loss function as the RMSE of price difference:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter |  |  |  |  |  |
| Initial Value | ﻿2.8187 | ﻿0.035 | ﻿0.0579 | ﻿0.0091 | ﻿-0.999 |
| Calibration Result | ﻿ ﻿2.81905739 | ﻿0.00206957183 | ﻿0.0479059218 | ﻿0.0166433728 | ﻿0.998501817 |
| Bounds | (0.001, 5) | (0.001, 5) | (0.001, 5) | (0.001, 5) | (-0.999, 0.999) |

1. Loss function as the RMSE of percentage price difference:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter |  |  |  |  |  |
| Initial Value | ﻿2.8187 | ﻿0.035 | ﻿0.0579 | ﻿0.0091 | ﻿0.999 |
| Calibration Result | ﻿ ﻿ ﻿2.81905646 | ﻿0.00223203112 | ﻿ ﻿0.0479316905 | ﻿﻿0.0166649264 | ﻿﻿0.998503128 |
| Bounds | (0.001, 5) | (0.001, 5) | (0.001, 5) | (0.001, 5) | (-0.999, 0.999) |

1. Loss function as the RMSE of volatility difference:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter |  |  |  |  |  |
| Initial Value | ﻿2 | ﻿0.1 | ﻿0.1 | ﻿0.1 | ﻿0.999 |
| Calibration Result | ﻿ ﻿ ﻿2.81869666 | ﻿ ﻿0.03444719 | 0.05798171 | ﻿0.0166433728 | ﻿0.998501817 |
| Bounds | (0.001, 5) | (0.001, 5) | (0.001, 5) | (0.001, 5) | (-0.999, 0.999) |

1. Loss function as the RMSE of percentage volatility difference:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter |  |  |  |  |  |
| Initial Value | ﻿2 | ﻿0.1 | ﻿0.1 | ﻿0.1 | ﻿0.999 |
| Calibration Result | ﻿ ﻿ ﻿2.00974568 | ﻿﻿0.668163150 | ﻿0.00111277528 | ﻿﻿0.001 | ﻿﻿0.988178844 |
| Bounds | (0.001, 5) | (0.001, 5) | (0.001, 5) | (0.001, 5) | (-0.999, 0.999) |

All of the calibration result seems to suggest that the two stochastic process are very much correlated with one process having a relatively big coefficient on the drift term.

Within all calibrated results, the one with price percentage difference as the loss function, achieves the optimal pricing error (volatility) less than 0.3bps, on average. The average bid-ask spread for the given sample period is around 1bps. We further examined the stability of these parameters over time by using the same set of parameters to predict future swaption value (either in price or volatility) for the following 10 days.

Overall, though, it is not always optimal to predict the next day’s value, (in fact the largest MSE is observed the following day), rather than performing calibration on daily basis, predicting errors tend to decrease over time (within the 10-day window). Specifically, in the given sample, on average pricing errors jumped to the highest the following day, then reverted back to be within 2% in price difference, and 2bps in volatility.

MSE Price (in $)/Percentage Price Difference Over 10 Days

MSE Vol (in bps)/Percentage Vol Difference Over 10 Days

However, to further explore the stability of the calibrated parameters requires sufficient market data and statistic assumptions, which after all, is not within the scope of this research. At current stage, we drew no conclusions on parameter stability and its impact to pricing errors.

# Conclusion

To conclude on this research, we explored the mechanism of an advanced short-term interest rate model, the G2++ and its implementation to pricing swaption. We have built a robust Python tool to conduct the daily calibration of the volatility surface. We calibrated the model parameters in order to minimize errors with market. The observed converging feature under this approach can be credited to the model’s ability to fit in more flexible volatility surface. However, this observation is not fully tested and whether predicting power exists requires further research. To the full scope of our research, we deemed our result to be positive and fulfilled the goal if it is to perform calibration in a daily basis.

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**Code on Github**:

<https://github.com/paragonhao/AFPSwaptionCalibration>